Inverse Queuing Model based Feedback Control for Elastic Container Provisioning of Web Systems in Kubernetes

Zhicheng Cai, Member, IEEE, Rajkumar Buyya, Fellow, IEEE,

Abstract—Container orchestration platforms such as Kubernetes and Kubernetes-derived KubeEdge (called Kubernetes-based systems collectively) have been gradually used to conduct unified management of Cloud, Fog and Edge resources. Container provisioning algorithms are crucial to guaranteeing quality of services (QoS) of such Kubernetes-based systems. However, most existing algorithms focus on placement and migration of fixed number of containers without considering elastic provisioning of containers. Meanwhile, widely used linear-performance-model-based feedback control or fixed-processing-rate based queuing model on diverse platforms cannot describe the performance of containerized Web systems accurately. Furthermore, a fixed reference point used by existing methods is likely to generate inaccurate output errors incurring great fluctuations encountered with large arrival-rate changes. In this paper, a feedback control method is designed based on a combination of varying-processing-rate queuing model and linear-model to provision containers elastically which improves the accuracy of output errors by learning reference models for different arrival rates automatically and mapping output errors from reference models to the queuing model. Our approach is compared with several state-of-art algorithms on a real Kubernetes cluster. Experimental results illustrate that our approach obtains the lowest percentage of service level agreement (SLA) violation and the second lowest cost.

Index Terms—Cloud, Fog and Edge Computing, Kubernetes, Container auto-scaling, Qos control, Queuing theory, Feedback control

1 INTRODUCTION

One of the most effective approaches to share resources of diverse systems such as private data centers, Cloud Computing, Fog and Edge Computing [1] among multiple applications is using containers which are more lightweight and portable than virtual machines (VMs) [2]. Kubernetes [3] is one of the popular container orchestrating systems and is gradually used to manage Cloud, Fog and Edge resources by containers seamlessly leading to many derivative platforms such as KubeEdge [4]. Kubernetes’s Pods (consisting of one or more containers) of different applications are deployed on VMs rented from public Clouds or physical machines (PMs) of Fog and Edge nodes. Meanwhile, Web applications and services (called Web systems collectively) are very common in Cloud, Fog and Edge Computing providing various functions to end users via different microservices deployed in the form of containers. One of the most crucial problems is to design container auto-scaling algorithms to control response time of each micro-service in Kubernetes-based platforms.

Most existing container provisioning algorithms focus on placement and migration of fixed number of containers on PMs or VMs rather than auto-scaling of containers [5], [6], [7]. However, the number of containers allocated to each micro-service has a great impact on request response times.

Kubernetes’s build-in auto-scaling scheduler [8] and most threshold-based methods [9] only add or remove container replicas based on resource usage rates which are indirect metrics for controlling response times. Therefore, the main goal of this paper is to design container auto-scaling methods for Kubernetes which adjust the number of containers allocated to each micro-service automatically to decrease resource consumption while guaranteeing quality of services (QoS). The main challenges of designing such auto-scaling algorithms include non-linear performance model of multi-container systems and finding appropriate output error computing methods.

Non-linear performance characteristics of Web systems make resource auto-scaling complex. QoS control has been studied extensively for traditional Web systems involving elastic provisioning of application resources, PMs or VMs. However, most existing methods belong to pure queuing-theory-based feed-forward control [10], [11], [12], [13] or linear-model-based feedback control [14], [15], [16] which lack feedback ability or cannot describe complex non-linear multi-container systems accurately. Although linear-model-based feedback control has been used to amend the inaccuracy of queuing models taking advantage of queuing and control theory together, the reference-point-derived linear performance model only works well when the system is near the reference point [17], [18].

The deviation between the reference point (e.g., reference response time) and the real-time response time is called output error which has a great influence on control performance. Selecting appropriate reference points to compute output errors is helpful to obtaining a stable control performance. In existing methods [19], reference points are usually
selected manually and kept unchanged for different arrival rates. However, a fixed reference point is likely to incur great control fluctuations because there are different stable working points for different arrival rates. Meanwhile, the inconsistency between the profiled performance model and sampled reference models is unavoidable which makes output errors mismatch with the profiled performance model leading to fierce fluctuations in some cases.

In this paper, an inverse-queuing-model-based feedback control method (Feedback_InverseQM) is proposed to guarantee the QoS of container-based Web systems in Kubernetes. The main contributions of our work are as follows.

1. A hybrid of varying-processing-rate-based queuing model and linear model is applied to describe the performance of the multi-container system more accurately. Inverse-queuing model is used to linearize the control system to simplify the controller design.

2. An online reference model learning method is designed to find appropriate reference points for different arrival rates increasing the accuracy of output errors.

3. An adaptive output-error-mapping method is proposed to amend the inconsistency between sampled reference models and the profiled performance model avoiding fierce control fluctuations.

The rest of this paper is organized as follows. Section 2 is the related work and Section 3 describes Web systems in Kubernetes. The proposed method is introduced in Section 4. Section 5 and 6 include performance evaluation on a real Kubernetes cluster, conclusions and future work.

2 RELATED WORK

Resource provisioning algorithms have been designed for Web systems in private data centers, Cloud, Fog and Edge Computing which mainly involve with allocation of application resources, PMs, VMs or containers.

2.1 Application Resource Provisioning

QoS control is one of the most important objectives of building self-adaptive computing systems [20], [21]. QoS control of traditional Web applications deployed on a single or multiple PMs usually involves application resources like Web server processes, sessions or database connections [22]. Feedback control is an essential method for application resource allocation problems. Linear-performance-model-based fixed gain [14], [15], adaptive [16] or multi-model switching [23] feedback control methods have been proposed for application resource auto-scaling.

Queuing models describe relations among the average response time, the request arrival rate and allocated resources of Web systems more accurately than linear models. Therefore, queuing models have been used to improve the performance of feedback control. For example, feedback control was used by Sha et al [17] and Xu et al. [18] to minimize residential errors produced by queuing models. Linear models, derived from queuing models which describe the linear relation of output-error changes and the adjustment of allocated resources near the reference point, are usually used to design feedback controllers. However, queuing-model-derived linear models only work well near reference points.

2.2 VM and PM Provisioning

VM placement, migration and auto-scaling are three important VM provisioning problems in Cloud, Fog and Edge Computing. The main objective of VM placement and migration is to decrease power consumption, total network latency [24] or improve resource utilization by placing or migrating a fixed number of VMs appropriately. For example, a fixed number of VMs are migrated among Fog Cloudlets to trace their mobile users [24] which only focuses on network latency affected by geography locations of VMs. Game theory is applied to deploy fixed number of VMs on Cloud and Edge hosts in terms of energy, performance and cost [25]. VM auto-scaling algorithms usually aim to minimize resource consumption by adjusting the number of allocated VMs to each application while guaranteeing QoS. Auto-scaling algorithms are the basis of VM placement and migration algorithms for improving the QoS collaboratively.

Threshold, reinforcement learning, queuing-and-control-theory-based methods have been widely used for VM auto-scaling problems. For threshold-based method, it is very tricky to set thresholds and control strength [20], [26]. Q-table and deep-neural-network based reinforcement learning methods proposed by Li et al. [27], Barrett et al. [28] and Tesauro et al. [12] have been used to allocate VMs or PMs to Web systems elastically. However, both Q-table and neural-network based methods need long training periods [26]. M/M/1 [12], M/M/N [10], [11], heterogeneous M/M/N [29] queuing models and queuing networks [13] have been used to determine the minimum number of VMs or PMs for guaranteeing QoS of Web systems. Nonetheless, pure queuing-model-based methods lack the ability of reacting to real-time output errors. Therefore, M/M/1-model-based feedback controller is designed to adjust the arrival-rate adjustment coefficient of a loosely coupled M/M/N model [30] based on output errors. However, this method is tailored for avoiding over-control incurred by interval-based charging models of Cloud VMs. For guaranteeing QoS of stream applications, VMs are allocated by comparing the real-time queuing length with the Little’s Law determined queuing length [31]. However, the number of VMs adjusted each time is determined by experience.

2.3 Container Provisioning

Container deployment, migration and auto-scaling problems considering load balancing, power consumption or QoS are becoming increasingly common on diverse underlying resources such as Cloud VMs [6], Fog Cloudlets [7], Edge nodes [32] and private data centers [9]. The number of containers allocated to each application are fixed or variable, and the total capacity of underlying resources is fixed or elastic too.

Most existing works focus on deploying and migrating a fixed number of containers on private or elastically rented underlying resources which can be modeled as bin-packing problems [7] and solved by CPLEX [6] or heuristic methods such as best fit decreasing bin packing (BFD) and time-bin BFD [5], [33]. These bin-packing methods are usually combined with VM auto-scaling algorithms such as cost-efficiency-based greedy method [5], [33] to implement container deployment on elastically rented VMs. Reinforcement
learning is another method to deploy and migrate contain-
ers considering geography locations of Fog nodes and user mobility [7]. Game theory is also used to schedule batch tasks to a fixed number of containers and deploy these containers on Edge hosts to minimize energy consumption and SLA violations [32]. For Web systems, response times are usually not only affected by geography distances, but also the number of deployed container replicas. However, auto-scaling of containers is not considered in these works.

Container auto-scaling algorithms mainly include workload prediction based proactive methods [34], threshold [9], [35] and control theory [19] based reactive methods. Time series analysis and machine learning methods have been used to predict workloads based on which containers are auto-scaled [34]. However, proactive methods lack the ability of reacting to real-time errors and workloads are not predictable for some cases. Some other works set a threshold to each application regarding CPU, Memory or performance metrics. One container is added when existing resources have been used up [9] or a fixed number of containers are added when the real-time response time is larger than a threshold [36]. Genetic algorithms [35] and linear program-
ing [37] have been used to find the optimal container distribution policy among multiple applications assuming that each application has a performance-degrading resource threshold or the utility of provisioning different numbers of containers to each application has been profiled in advance. The difficulty of threshold-based methods is how to select appropriate threshold values suitable for different arrival rates [34]. An inverse proportional performance model, which is more accurate than linear models, was used to design a feedback control method by Baresi et al. [19] to allocate containers to Web applications with fixed reference point of 0.9 × SLA.

2.4 Comparison with Existing Algorithms

Table 1 shows a comparison of our approach with existing resource provisioning algorithms. Firstly, linear or queuing-model-derived linear models used in most existing feedback control methods cannot describe the container based sys-

們 accurately [38]. On the contrary, our approach applies a feedback controller based on a more accurate performance model which is the hybrid of varying-processing-rate M/M/N model and linear model. Secondly, most existing works use a fixed reference point obtained by experience to compute output errors directly. However, appropriate reference points are different for various request arrival rates. Therefore, our approach applies an automatic reference point identification method and an adaptive output error mapping method to improve the control stability. Finally, performance of many container auto-scaling algorithms is only evaluated on simulation platforms while our approach is implemented as a user-level scheduler of real Kubernetes-based platforms.

3 WEB SYSTEMS IN KUBERNETES

It is flexible and inter-operable to organize diverse Web systems using micro-service-based architecture as shown in Figure 1. Each tier of such Web systems can be implemented as a micro-service which runs in parallel containers to support large-scale requests. Each micro-service provides a single function and is usually encapsulated as a RESTful or SOAP-based Web service. Since Kubernetes manages re-
sources in the form of Pods [3], containers of micro-services are embedded in Pods and deployed on PMs or VMs of Cloud, Fog and Edge Computing which are managed by Kubernetes-based systems uniformly. It is very common that one Pod only contains one container. Pods with different colors in Figure 1 consist of containers of different micro-

services. User requests of the same micro-service are dis-
buted to containers using different balancing algorithms by nginx-ingress-controllers [41]. Real-time performance of each micro-service is monitored through Kubernetes java-client-interface [42]. Based on collected performance metrics, the container Auto-scaling Controller (ASC) is imple-
mented as a user-level plugin to allocate appropriate num-

 bers of containers (Pods) to each micro-service separately. Although geography locations of containers influence the response time either, it is assumed that containers with the same configuration on Cloud and Edge hosts have the same performance. The auto-scaling considering geography locations will be considered in future works. Each micro-

service has its own ASC, i.e., a decentralized controlling method is applied [19].

The average response time and resource cost are two crucial metrics of containerized Web systems. In the SLA of a micro-service, it is usually defined that k% of response times should be smaller than a given threshold $W_{sla}$. Because identical containers are usually required by the same micro-

service, one Container Unit (CU) is defined to be the cost of allocating one container to a micro-service in one control interval. The objective of this article is to design container auto-scaling algorithms for ASCs to decrease the number of consumed CUs while guaranteeing SLAs.

4 PROPOSED FEEDBACK CONTROL METHOD

In this paper, an Inverse-Queuing-Model-based feedback control method (Feedback_InverseQM) is proposed. A hy-
TABLE 1
Comparison of our approach with existing resource provisioning algorithms for Web Systems

<table>
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<td>A single Web Server</td>
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<td></td>
<td></td>
<td>Queuing-model-derived linear model based hybrid control</td>
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<tr>
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<td>Inverse-M/M/N-model based feedback control</td>
<td>Inverse-M/M/N-model based feedback control</td>
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Let $\lambda$ be the request arrival rate and $N$ be the number of containers. The processing rate of each container is

$$\mu = \mu_b + c/\lambda$$

(1)

where $\mu_b$ is the basic processing rate and $c$ is the inverse-proportional coefficient of $\lambda$. Larger arrival rates mean smaller processing rates. According to M/M/N model, the probability of no requests in the whole system is

$$P_0 = \left( \frac{\lambda}{\mu} \right)^N \sum_{k=0}^{N-1} \frac{\lambda^k}{k!} + \frac{\lambda^N}{N!(1 - \frac{\lambda}{\mu})\mu^N} \right)^{-1}$$

(2)

The expectation of the number of requests in the waiting queue and under processing is

$$L_s(N, \lambda, \mu) = \frac{\lambda^N}{N!(1 - \frac{\lambda}{\mu})^2} P_0 + \frac{\lambda^N}{\mu}$$

(3)

The expectation of the average response time is

$$W_s(N, \lambda, \mu) = \frac{L_s(N, \lambda, \mu)}{\lambda}$$

(4)
The average response time $y_k$ of control step $k$ is set to be a weighted combination of the past value $y_{k-1}$ and M/M/N queuing model with varying processing rates as

$$y_k = a 	imes y_{k-1} + (1-a) 	imes W_s(N, \lambda, \mu)$$

(5)

To obtain values of parameters $a$, $\mu_b$, and $c$, the least square method is applied based on historical data collected from Kubernetes platforms under $0 < a < 0.3$, $\mu_b > 0$ and $c > 0$ constraints.

### 4.2 Automatic Reference-Model Learning

For container-based queuing systems, the given $W_{sla}$ might be far from response times of stable working points. Appropriate reference response times smaller than $W_{sla}$ should be found to compute output errors which is the basis of feedback control rather than using $W_{sla}$ directly. Because request arrival rates have a great impact on stable points, different reference points should be found for different arrival rates. An automatic reference-model learning method (ARML) is proposed to profile reference models for different arrival rates based on samples in the form of $s = (n_s, \lambda_s, y_s)$ in which $n_s$ is the container number, $\lambda_s$ is the arrival rate and $y_s$ is the average response time. For model $m$ with reference arrival rate $\lambda_m$, a reference response time $W^{r}_{m}$ and a lower bound of container-number $n^{r}_{m}$ will be studied from samples. A reference-model set $M$ is used to store studied models and new models are gradually added. In order to decrease the number of studied reference models, the difference between any two models’ reference arrival rates should be larger than a gap $g$ (e.g., 5/s). Samples with similar arrival rates are used to profile the same model.

Formal description of ARML is shown in Algorithm 1. In auto-scaling step $k$, a new sample $s$ will be collected and a corresponding reference model $m_k = \arg \min_{m \in M} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g \}$ is tried to be found. If $m_k = \text{null}$, a new model $m_k$ with reference arrival rate $\lambda$ is created and added to $M$. Otherwise, $s$ will be added to $m_k$ only if $m_k = m_{k-1}$. In each model, samples are stored in a hashmap $h_{m_k} = \langle n, bt_n \rangle$ where container number $n$ is the key and bucket $bt_n$ contains average response times of samples with container number $n$. For each $n$, only latest 10 average response times are stored in $bt_n$ and values with distances larger than $0.4 \times \text{average}(bt_n)$ from the average value $\text{average}(bt_n)$ are filtered. Then $\text{average}(bt_n)$ is updated based on filtered values for each $n$ and added to a set $Y$. Finally, the $i$-th (e.g., third) largest value in $Y$ is taken as the reference response time $W^{r}_{m_k}$. When $y_s > W_{sla}$, $n_s$ is used to update the lower bound of container number $n^{r}_{m_k}$ if $n_s < n^{r}_{m_k}$ or $n^{r}_{m_k} = \text{null}$. Whenever a new sample $s$ is collected from the system, $W^{r}_{m_k}$ and $n^{r}_{m_k}$ are updated as mentioned above. Only when $n^{r}_{m_k}$ has been assigned a value and the number of hashmap’s keys ($h_{key}$) is larger than 4, the model $m_k$ is called mature and added to a mature model set $M'$.

After $s$ is added to $m_k$, a mature reference model $m'_k = \arg \min_{m \in M'} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g \}$ is tried to be found. $g$ is usually set to be larger than $g$ (e.g., 20/s) to allow mature models to guide more scenarios with larger arrival-rate differences. $m_k = m'_k$ only when $m_k$ is mature. If $m_k$ can be found, $W^{r}_{m_k}$ is selected as the reference response time which will be used as the reference point of feedback control. Otherwise, a sampling method is activated to collect more samples for $m_k$. Let $n^{a}_{m_k}$ and $n^{l}_{m_k}$ be the smallest and largest keys of $h_{m_k}$. If $n^{a}_{m_k} - 1 > n^{r}_{m_k}$ or $n^{r}_{m_k} = \text{null}$, container number $N_k$ of current step $k$ is set to be $n^{a}_{m_k} - 1$. Otherwise, $N_k = n^{a}_{m_k} + 1$.

### 4.3 Inverse Queuing Model based Controller Design

If a mature reference performance $m'_k$ can be found, feedback control is applied to follow the reference time $W^{r}_{m_k}$.

The control error is

$$e_k = W^{r}_{m_k} - y_k$$

(6)

A controller should be designed based on performance model in Equation (5) and $e_k$. However, designing a feedback controller based on the hybrid non-linear performance model directly is very complex. Therefore, an inverse function of queuing model is used to linearize the performance model which simplifies the design of feedback controllers [19], [30] as shown in Figure 2. Because it is complex to deduce an inverse-queuing function from Equation (4) directly, an exhaust search method is used to implement the inverse-queuing function to find the corresponding $N_k$ given $u_k$ as shown in Algorithm 2. The queuing function from $N_k$ to $u_k$ (queuing model part) in the performance model is counteracted by the inverse-queuing function from $u_k$ to $N_k$. In other words, $u_k = u_{st}$, if queuing model part is accurate in describing multi-container systems. Then the linearized performance model is

$$y_k = a \times y_{k-1} + (1-a)u_{k-1} = a \times y_{k-1} + (1-a)u_{k-1}$$

(7)

and the Z-transfer function of the performance model is

$$\frac{Y}{U} = \frac{1-a}{z - a}$$

(8)
**Algorithm 1: Automatic Reference Model Learning (ARML)**

**input**: $s, \lambda, \mu, k_{-1}, W_{sla}$
1. Initialize $i \leftarrow 3, g \leftarrow 5/s, g' \leftarrow 20/s, Y \leftarrow \emptyset$;
2. $m_k \leftarrow \arg \min_{m \in M} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g' \}$;
3. if $m_k = \text{null}$ then
   4. Create a new model $m_k$;
   5. $\lambda_{mk} \leftarrow \lambda, M \leftarrow M \cup \{m_k\}$;
4. if $m_k = m_{k-1}$ then
5. Store $s$ in hashmap $h_{mk}$;
6. foreach $n \in h_{mk}^r$ do
   7. Calculate $\text{average}(bt_n)$;
   8. $Y \leftarrow Y \cup \{ \text{average}(bt_n) \}$;
   9. $W_{m_k}^r \leftarrow$ the $i$-th largest value in $Y$ ;
10. if $y_s > W_{sla}$ and $(n_s < n_{mk}^r$ or $n_{mk} = \text{null})$ then
   11. $n_{mk}^r \leftarrow n_s$;
   12. if $n_{mk}^r \neq \text{null}$ and $|h_{mk}^r| > 4$ then
      13. $M' \leftarrow M' \cup \{m_k\}$;
16. $m_k = \arg \min_{m \in M'} \{ \lambda_d = |\lambda_m - \lambda|, \lambda_d < g' \}$;
   17. if $m_k = \text{null}$ then
      18. if $n_{mk}^r - 1 > n_{mk}^r$ or $n_{mk}^r = \text{null}$ then
         19. $N_k \leftarrow n_{mk}^r - 1$;
      20. else
         21. $N_k \leftarrow n_{mk}^r + 1$;
      22. return null, $N_k$;
   23. else
      24. return $m_k$, null;

**Algorithm 2: Inverse Queuing Model (InverseQM)**

**input**: $\lambda, \mu, u_k$
1. $N_k \leftarrow \lceil \frac{N_k}{m_k} \rceil$
2. while $W_s(N_k, \lambda, \mu) > u_k$ do
   3. $N_k \leftarrow N_k + 1$;
4. return $N_k$

Z-transfer function of the whole feedback system is

$$F_R(z) = Y \frac{1-a}{z-a} \frac{K_I z}{z-a \frac{1}{z} + rac{1}{z} + 1}$$

$$(11)$$

Through linearization, queuing theory and feedback control are combined to provide accurate performance modeling and feedback abilities simultaneously.

**4.4 Adaptive Output Error Mapping Method**

In order to keep the system stable, $u_k$ should be smaller than $W^* = W_s(n_{mk}^r, \lambda, \mu)$ to make $N_k$ larger than $n_{mk}^r$. Meanwhile, $u_k$ cannot be smaller than $W^d = W_s(N_m, \lambda, \mu)$ to allocate no more than the maximum number of available containers $N_m$. Because $n_{mk}^r$ is determined by one sample and may be not accurate. To allow sampling on $n_{mk}^r$ again, $W^*_s$ is set to be

$$W^*_s = \frac{W_s(n_{mk}^r, \lambda, \mu) + W_s(n_{mk}^r - 1, \lambda, \mu)}{2}$$

The output error $e_k$, which is calculated based on reference model $m_k$, will be used to adjust $u_k$ of the queuing model by Equation (9). However, there are still unavoidable deviation between reference model $m_k$ and profiled performance model as shown in Figure 3. When the deviation is too large, $u_k$ is likely to change greatly and exceed the lower or upper bounds $[W^d_{m_k}, W^u_{m_k}]$ of the queuing model part. Let $[W^d_{m_k}, W^u_{m_k}]$ be the lower and upper bounds of reference model $m_k$ which are the smallest and largest response times smaller than $W_{sla}$, respectively. To guarantee the bounds $[W^d_{m_k}, W^u_{m_k}]$, an adaptive output-error mapping method (AOM) is proposed to map $e_k$ from the reference model space to $e_k$ in the queuing model space.

**Theorem 1.** If $e_k$ is mapped to $e'_k$ in proportional, there is

$$e'_k = e_k \times K_a$$

$$K_a = \begin{cases} 
     \frac{y_k - W^d_{m_k}}{W^u_{m_k} - W^d_{m_k}} & \text{if } y_k > W^u_{m_k} \\
     \frac{W^d_{m_k} - y_k}{W^d_{m_k}} & \text{otherwise}
\end{cases}$$

**Proof 1.** Let $u^r$ be the reference point in queuing model space which is the corresponding map of $W^r_{m_k}$ of reference model. When $y_k > W^u_{m_k}$, $e'_k = u_k - u^r$. If mapping is done in proportion, the ratio of $e_k$ to $y_k - W^d_{m_k}$ is equal to the ratio of the output error to the maximum decreasing range of response times in the

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**Fig. 2. Architecture of hybrid control for container-based Web Systems**

An integral feedback controller is designed based on the linear part merely ignoring the queuing model part as follows

$$u_k = u_{k-1} + K_I \times e_k \quad (9)$$

Z-transfer function of the integral controller is

$$U \frac{K_I z}{E} = \frac{z - 1}{z - 1}$$

$$\quad (10)$$
proposed performance model, then equal to the ratio of $e'_k$ to $u_{k-1} - W_s'$ of the queuing model part as follows

$$
y_k - W_s' = (a \times y_{k-1} + (1-a)u_{k-1}) - (a \times y_{k-1} + (1-a)u') = (a \times y_{k-1} + (1-a)u_{k-1} - (a \times y_{k-1} + (1-a)W_s')) = (1-a)u_k - W_d' = e'_k / (u_{k-1} - W_s') \tag{15}
$$

When $y_k \leq W_r'$, the prove is similar.

In order to avoid large fluctuations, $K_a$ is trimmed to 1 when $K_a > 1$. By substituting Equation (13) into Equation (9), the integral controller becomes

$$
u_k = \alpha \times y_k + \beta \times u_k - \gamma \times e_k \tag{16}
$$

where $K_i' = K_i \times K_a$ is called adaptive control gain.

### 4.5 Queuing-length-based Unstable State Scheduling

When the arrival rate $\lambda$ is larger than the total processing ability $\mu$, the system is unstable and Equations (2), (3) and (4) are not valid. Therefore, it is not suitable to determine $N_k$ based on $e_k$ directly. In unstable states, the real queuing length $L_r$ is larger than $L_s(N_k, \lambda, \mu)$ and increases continually until the allowed maximum number of waiting connections is reached, i.e., queuing lengths represent the blocking degrees. Therefore, a queuing-length-based unstable state provisioning method is applied as shown in Algorithm 3. In lines 1 to 5, the minimum number of containers $N$ fulfilling $W_{sla}$ according to M/M/1 queueing model is first obtained. Then $N_k$ should be larger than $N_{k-1}$ and not smaller than $N$ in lines 6 to 8. Next $N_k$ is increased one by one to make sure that $L_r$ can decrease to $L_s$ in $T_r$ (e.g., 10) seconds by increasing $N_k - N_{k-1}$ containers. Finally, $u_k$ is updated to guarantee that $N_k$ containers are still rented in the next control step if there is no output errors.

### 4.6 Formal Description of Feedback_InverseQM

Formal description of Feedback_InverseQM is shown in Algorithm 4. At first, performance model is profiled using historical data. Then QLP is invoked, if the real queuing length $L_r$ is larger than $\alpha$ (e.g., 10) times of the theory queuing length $L_s(N_k, \lambda, \mu) > y_k$ is larger than $\beta$ (e.g., 1.2) times of the largest stable response time $W_s(N, \lambda, \mu)$. Otherwise, ARML is called to get a reference model $m_k$ or obtain a sampling $N_k$. If $m_k \neq null$, Equation (16) is used to obtain $u_k$, and InverseQM is applied to get the real control action $N_k$. Finally, the number of containers allocated to the Web system is adjusted to be $N_k$.

### Algorithm 3: Queuing-length-based provisioning (QLP)

```
input : \lambda, \mu, W_{sla}, N_{k-1}, L_r
1 N ← \lceil \frac{\lambda}{\mu} \rceil, T_r ← 10;
2 W_s' ← a \times y_{k-1} + W_s(N, \lambda, \mu) \times (1-a);
3 while $W_s' > W_{sla}$ do
4 \quad N ← N + 1;
5 \quad W_s' ← a \times y_{k-1} + W_s(N, \lambda, \mu) \times (1-a);
6 \quad N_k ← N_k + 1;
7 \quad if $N_k < N$ then
8 \quad \quad N_k ← N
9 \quad while $\mu \times (N_k - N_{k-1}) \times T_r < L_r - L_s(N_k, \lambda, \mu)$ do
10 \quad \quad N_k ← N_k + 1;
11 \quad if $(N_k - 1) \times \mu > \lambda$ then
12 \quad \quad u_k ← W_s(N_k, \mu) + W_s(N_k-1, \lambda, \mu);
13 \quad else
14 \quad \quad u_k ← W_s(N_k, \lambda, \mu) + \epsilon
15 return $N_k$
```

### Algorithm 4: Inverse queuing model based feedback control (FeedBack_InverseQM)

```
input : \lambda, L_r, y_k
1 Initialize $\alpha \leftarrow 10, \beta \leftarrow 1.2$;
2 $\mu_b, c, \alpha \leftarrow $ Profiling the performance model;
3 $\mu = \mu_b + c/\lambda, N \leftarrow \lceil \frac{\lambda}{\mu} \rceil$;
4 if $L_r > L_s(N_k, \lambda, \mu) \times \alpha$ or $y_k > W_s(N, \lambda, \mu) \times \beta$ then
5 \quad $N_k \leftarrow$ QLP($\lambda, \mu, W_{sla}, N_{k-1}, L_r$);
6 else
7 \quad $m_k', N_k \leftarrow$ ARML($s, \lambda, m_{k-1}, W_{sla}$);
8 \quad if $m_k \neq null$ then
9 \quad \quad $u_k \leftarrow$ Equation (16);
10 \quad \quad $N_k \leftarrow$ InverseQM($\lambda, \mu, u_k$);
11 Allocate $N_k$ containers to the system;
```

### 5 Performance Evaluation

Our proposed FeedBack_InverseQM is implemented as a user-level scheduler for Kubernetes. It is compared with existing algorithms on a real Kubernetes cluster which locates on four physical machines with the configuration of 6~12
virtual CPU cores and 8~16 GB Memory. The Kubernetes cluster consists of one Master and four Worker nodes. A service for calculating Fibonacci numbers is adopted as the test-bed and the input of the service is the length of the generated Fibonacci series which is selected from the interval [26, 33] for each request randomly. Requests of the service are redirected to different containers by the nginx-ingress-controller using exponentially weighted moving average (EWMA) as the load-balancing algorithm [41]. The connection timeout time is set to 10 seconds and the allowed maximum number of connections is 500 per container. The user access traces of Wikipedia [43] are considered as shown in Figure 4 with common peaks and valleys of Web systems are used to generate requests through JMeter [44].

FeedBack_InverseQM is first compared with FeedBack_QMDL which is a classical feedback control method for QoS control based on queuing-model-derived linear models [17], [18]. Although FeedBack_QMCA [30] is tailored for hourly-priced VM provisioning, FeedBack_InverseQM is still compared with FeedBack_QMCA by removing the VM-releasing status checking. Finally, FeedBack_InverseQM is also compared with FeedBack_InverseP [19] which is one of the elastic container provisioning algorithms considering QoS control. Average response times of requests obtained from logs of nginx-ingress-controllers every minute and the total consumed CUs are two adopted metrics of algorithm comparison. The length of control interval of all algorithms is 250 seconds. Since FeedBack_InverseQM has an initial sampling period, the cost of each compared algorithm is only the accumulation of consumed CUs after the initial 500 minutes for fair comparison.

5.1 Parameter Tuning

The control gain $K_I$ determines the poles of the system which has a great impact on the settle time and overshoot. Poles can be derived by setting $z^2 + [(1-a)K_I - 1-a]z + a = 0$ given $K_I$. For example, poles $p_1 = 0.44 + 0.32619013j$ and $p_2 = 0.44 - 0.32619013j$ when $K_I = 0.6$. According to root locus which draws the figure of poles as $K_I$ changes [22], larger $K_I$ usually means larger overshoots, and too small or too large $K_I$ are likely to incur long settle time. A set of candidate values $S_{K_I} = \{0.05, 0.1, 0.15, 0.3, 0.6\}$ are selected based on root locus. Then the real performance of $K_I \in S_{K_I}$ is evaluated by experiments. Figures 5 and 6 show consumed container numbers and average response times of FeedBack_InverseQM with different $K_I$ which illustrate that container numbers of FeedBack_InverseQM changes more quickly as $K_I$ increases in total. For smaller $K_I$, FeedBack_InverseQM reacts to excess containers and SLA violations very slowly. For example, excess containers cannot be released in time consuming the most cost (2454 CUs) when $K_I = 0.05$ and SLA is violated for many periods when $K_I = 0.1$. On the contrary, for larger $K_I$, the container number fluctuate fiercely leading to more SLA violations when $K_I \geq 0.3$ and extremely large $K_I = 0.6$ even incurs the second most cost (2290 CUs) because of frequent container allocating and deallocating. Therefore, $K_I = 0.15$ is finally selected which obtains the most appropriate control strength leading to fewer SLA violations and a lower cost simultaneously.

According to the performance tuning results of [30], poles of Feedback_QMDL and Feedback_QMCA are set to
be 0.9 and 0, respectively. The pole of Feedback_InverseP is set to be 0.95 consistent with [19]. Parameters $\mu_k = 7.771$, $c = 1574.510$ and $a = 0.215$ of the varying-processing-rate-based performance model were obtained based on the given historical data which may change over time. Parameters of the fixed-processing-rate-based performance model can be acquired similarly by setting $c = 0$. Figure 7 shows real response times and estimated values of profiled performance models which illustrates that the proposed varying-processing-rate-based method (blue dots) describes the system more accurately than the fixed-request-processing-rate-based method (green dots).

### 5.2 Experimental Results

Table 3 shows percentages of SLA violations and costs of compared algorithms which illustrate that proposed FeedBack_InverseQM obtains the lowest percentage of SLA violation with the second lowest cost (2250 CUs) in total.

![Fig. 7. Real response times and estimated values of profiled performance models using fixed and varying processing rates, respectively](image)

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>SLA Violations</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feedback_InverseQM</td>
<td>2.36%</td>
<td>2250 CUs</td>
</tr>
<tr>
<td>Feedback_QMDL</td>
<td>10.80%</td>
<td>2289 CUs</td>
</tr>
<tr>
<td>Feedback_QMCA</td>
<td>21.26%</td>
<td>1989 CUs</td>
</tr>
<tr>
<td>Feedback_InverseP</td>
<td>52.99%</td>
<td>2487 CUs</td>
</tr>
</tbody>
</table>

Table 3: Percentages of SLA violations and Costs

Figure 8 shows container numbers and response times of FeedBack_InverseQM which denotes that most response times are smaller than $W_{ref}$. Reasons of FeedBack_InverseQM’s best performance are as follows. Firstly, ARML of FeedBack_InverseQM is helpful to improve the accuracy of output errors. Because different arrival rates have different stable working points, samples are collected by ARML to study reference response times for diverse arrival rates leading to some fluctuations in the initial stage of Figure 8. Studied reference response times are shown in Figure 9 which increases the accuracy of output errors. Secondly, AOM of FeedBack_InverseQM is able to improve control stability. FeedBack_InverseQM is sensitive to output errors when $u_{k-1}$ is near relatively flatten parts of the queuing model as shown in Figure 3. Flatten parts of performance models reflect the nature of queuing systems which enable controllers react to SLA violations quickly. However, there are unavoidable deviations between the reference model and the profiled queuing model because of arrival rate differences and inaccuracy of queuing models. Sometimes, the original output errors generated by the reference model are so large that it makes $u_k$ fluctuate drastically. Figure 10 shows adaptive control gains of different steps generated by AOM which are used to map output errors from the reference model to the queuing model in proportion to avoid fierce fluctuations. Both ARML and AOM make

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**Fig. 7.** Real response times and estimated values of profiled performance models using fixed and varying processing rates, respectively

**Fig. 8.** Container numbers and response times of FeedBack_InverseQM

**Fig. 9.** Reference response times of control steps with mature models
FeedBack_InverseQM obtain the most stable performance as shown in Figure 8 except the initial sampling stage.

Figure 11 illustrates that FeedBack-QMDL reacts slowly to fast arrival-rate changes leading to SLA violations or delaying the release of excess containers. The reason is that the output container number of FeedBack-QMDL is the plus of M/M/N model’s output and an additional value determined by feedback control. Meanwhile, different arrival rates need quite different additional values, and the changing speed of the additional value cannot meet the requirement when the arrival rate changes quickly. However, the changing speed of the additional value cannot be increased by setting smaller poles (e.g., 0.6) anymore, because it is likely to allocate or release excess containers incurring more SLA violations or higher costs for scenarios with slow arrival rate changing speeds. The main reason is that the changing speed of the additional value is fixed and linear to the output errors without considering various arrival rates with different changing speeds.

Figure 12 demonstrates that FeedBackQMCA is very likely to allocate or dealllocate excess containers leading to frequent large fluctuations (larger overshoots) at periods with slow arrival-rate changing speeds of which the reasons are as follows. In FeedBackQMCA, the inaccuracy of queuing model is fixed by an arrival-rate adjustment coefficient. Experimental results illustrate that different arrival rates need quite different adjustment coefficients. When the arrival rate changes quickly, it takes a long time to adjust the coefficient leading to SLA violations or higher costs given small control gains. Therefore, the current proportional control gain has been increased as large as possible to speed up the changing speed of the adjustment coefficient to meet the requirement of fast arrival rate changes. However, the controller gain, which fulfills the fast-changed arrival rates, makes the system fluctuate when arrival rates change slowly. It is hard to find an appropriate gain suitable for different arrival-rate changing speeds. Meanwhile, a fixed reference point is not able to generate suitable output errors for all arrival rates which misleads the controller to release or rent excess containers incurring SLA violations or higher costs.
Designing container auto-scaling algorithms considering geography distribution of Cloud, Fog and Edge resources is promising future work.

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Zhicheng Cai received his Ph.D. degree in Computer Science and Engineering from South-east University, China, in 2015. He is an associate professor at Nanjing University of Science and Technology, China. He is also a visiting scholar of the University of Melbourne from Sep 2019 to Sep 2020. His research interests focus on resource scheduling of Web systems and batch tasks in Cloud, Fog and Edge Computing.

Rajkumar Buyya is a Redmond Barry Distinguished Professor and Director of the Cloud Computing and Distributed Systems (CLOUDS) Laboratory at the University of Melbourne, Australia. He has authored over 625 publications and seven text books including “Mastering Cloud Computing”. He also edited several books including “Cloud Computing: Principles and Paradigms”. He is one of the highly cited authors in computer science and software engineering worldwide (h-index=137, g-index=304, 100,700+ citations). Microsoft Academic Search Index ranked Dr. Buyya as #1 author in the world (2005-2016) for both field rating and citations evaluations in the area of Distributed and Parallel Computing. A Scimetic Analysis of Cloud Computing Literature by German scientists ranked Dr. Buyya as the World’s Top-Cited (#1 Author and the World’s Most-Productive (#1) Author in Cloud Computing. He is recognized as a “Web of Science Highly Cited Researcher” in 2016, 2017, and 2018 by Thomson Reuters, a Fellow of IEEE, and Scopus Researcher of the Year 2017 with Excellence in Innovative Research Award by Elsevier for his outstanding contributions to Cloud computing. He is currently serving as Co-Editor-in-Chief of Journal of Software: Practice and Experience.